1 (1) 8' $N = \frac{S}{(2\pi)^2} \cdot 2 \int_{k<k_F} d^2k = \frac{S}{(2\pi)^2} 2\pi k_F^2$

33'

$\therefore \quad n = \frac{N}{S} = \frac{k_F^2}{2\pi}$

(2) 8' $\frac{1}{n} = \pi r_s^2 \qquad r_s = \sqrt{\frac{1}{n\pi}}$

$\therefore \quad k_F = \sqrt{2\pi n} = \frac{\sqrt{2}}{r_s}$ that is $k_F \cdot r_s = \sqrt{2}$

(3) total number of state, per unit volume whose energy is less than ε is:

8' $G(\varepsilon) = \frac{1}{S} \cdot N(\varepsilon) = \frac{1}{S} \cdot \frac{S}{(2\pi)^2} \cdot 2\pi k^2(\varepsilon) = \frac{k^2(\varepsilon)}{2\pi}$

$\because \quad \varepsilon = \frac{\hbar^2 k^2}{2m} \qquad k^2 = \frac{2m\varepsilon}{\hbar^2}$

$\therefore \quad G(\varepsilon) = \frac{m\varepsilon}{\pi\hbar^2}$

$D(\varepsilon) = g(\varepsilon) = \frac{dG(\varepsilon)}{d\varepsilon} = \frac{m}{\pi\hbar^2}$ is a constant $D(\varepsilon)$

9' (4) $n_0 = \frac{N}{V} = \int_0^\infty d\varepsilon\, D(\varepsilon) f_0(\varepsilon) = \int_0^\infty d(\varepsilon)\, f_0(\varepsilon) \frac{dG(\varepsilon)}{d\varepsilon}$

here $D(\varepsilon) = \frac{m}{\pi\hbar^2} = \text{constant}$ so $G(\varepsilon) = \frac{m\varepsilon}{\pi\hbar^2}$

In sommerfield expansion:

$G(\varepsilon) = G(\xi) + (\varepsilon-\xi)G'(\xi) + \frac{1}{2!}(\varepsilon-\xi)^2 G''(\xi) + \cdots$

that is a taylor expansion at the point of chemical potential

so $n_0 = G(\xi)\underbrace{\int_0^\infty d\varepsilon\left(-\frac{\partial f_0}{\partial \varepsilon}\right)}_{1} + G'(\xi)\underbrace{\int_0^\infty d\varepsilon\,(\varepsilon-\xi)\left(-\frac{\partial f_0}{\partial \varepsilon}\right)}_{\text{odd function} = 0} + \frac{1}{2!}\underbrace{G''(\xi)}_{0}\int_0^\infty d\varepsilon(\xi-\varepsilon)^2\left(-\frac{\partial f_0}{\partial \varepsilon}\right) + \cdots$

since $G^{(n)}(\xi) = 0$ for $n \geq 2$

so only the first term count in sommerfield expansion:

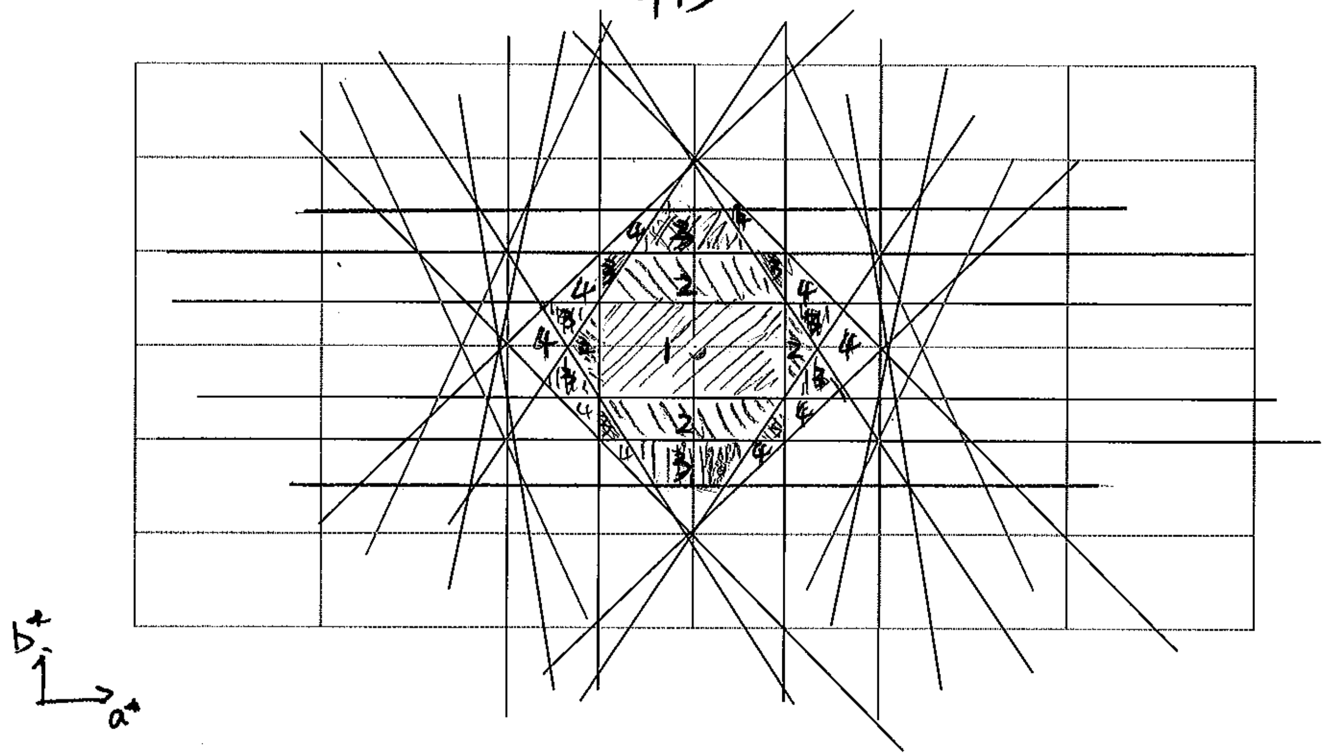
that is $n_0 = G(\xi) = \frac{m\xi}{\pi\hbar^2} \longrightarrow n = \frac{k_F^2}{2\pi} = \frac{m\varepsilon_F}{\pi\hbar^2}$

$\xi = \varepsilon_F$

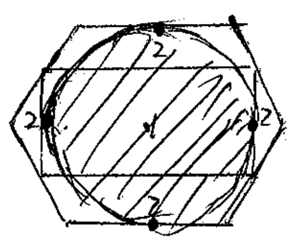
Chemical pontential equals fermi level at all temperature

2. 33

+15

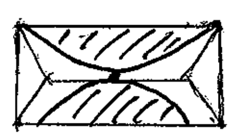


+10

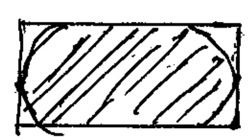


1st, 2nd, Zone

+8



2nd zone (reduced)



1st zone

fermi's phere

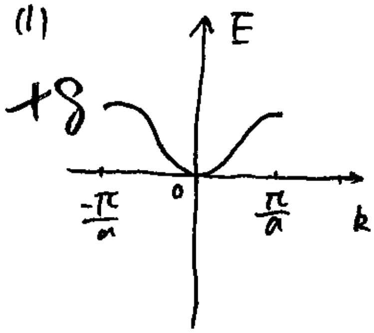
fermi's phere in 2nd Zone

fermisphere in 1st

the radius of fermi's phere is $r_F = \frac{1}{2}a^* = b^*$, just reach the boundary of 1st zo

33

3. (1)



top of band $k = \pm\frac{\pi}{a}$

bottom of band $k = 0$

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}$$

$$= \frac{1}{\hbar^2}\frac{\partial}{\partial k}\left[\frac{\hbar^2}{ma}(\sin ka - \frac{1}{4}\sin 2ka)\right]$$

$$= \frac{1}{ma}\frac{\partial}{\partial k}(\sin ka - \frac{1}{4}\sin 2ka)$$

$$= \frac{1}{m}(\cos ka - \frac{1}{2}\cos 2ka)$$

top $k = \pm\frac{\pi}{a} \Rightarrow \cos ka = -1 \quad \cos 2ka = 1 \qquad m^* = -\frac{2}{3}m$

bottom $k = 0 \Rightarrow \cos ka = \cos 2ka = 1 \qquad m^* = 2m$

(2) same as (1)

+8 $m_i^{*-1} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2} = \frac{1}{m}(\cos ka - \frac{1}{2}\cos 2ka)$

$$m_i = \frac{2m}{2\cos ka - \cos 2ka}$$

(3) $\vec{v}(k) = \frac{1}{\hbar}\vec{\nabla}_k \varepsilon_n(\vec{k}) \Leftrightarrow \frac{\hbar k}{m_v}$

+8 In this 1D case, $v = \frac{1}{\hbar}\frac{\partial}{\partial k}\left[\frac{\hbar^2}{ma^2}(\frac{7}{8} - \cos ka + \frac{1}{8}\cos 2ka)\right]$

$$= \frac{\hbar}{ma}(\sin ka - \frac{1}{4}\sin 2ka) \Rightarrow m_v = \frac{ak}{\sin ka - \frac{1}{4}\sin 2ka} m$$

~~$\approx \frac{\hbar}{ma}(ka - \frac{2ka}{4}) = \frac{1}{2m}$ when k is small~~

So $m_v \approx 2m$ when k is small $\to 0$

(4)

+9 In 1D, Density of state ~~$G(\varepsilon)$~~

$$g(k)dk = \frac{dk}{2\pi}$$

$$G(\varepsilon)d\varepsilon = g(k)\frac{dk}{dE}d\varepsilon$$

$\because \; dE = \frac{\hbar^2}{ma}(\sin ka - \frac{1}{4}\sin 2ka)\,dk$

$$\therefore \frac{dk}{dE} = \frac{ma}{\hbar^2(\sin ka - \frac{1}{4}\sin 2ka)}$$

$$\therefore G(\varepsilon)d\varepsilon = \frac{ma}{2\pi\hbar^2(\sin ka - \frac{1}{4}\sin 2ka)}d\varepsilon$$

Free electron 1D

$$E = \frac{\hbar^2 k^2}{2m}$$

$$\frac{dk}{dE} = \frac{m}{\hbar^2 k}$$

$$G(\varepsilon) = \frac{m}{2\pi\hbar^2 k}$$

$$\therefore m_d = \frac{(\sin ka - \frac{1}{4}\sin 2ka)}{ak} m$$

33

4. Equation of motion:

$$\begin{cases} \hbar \dot{\vec{k}} = -e\left(\vec{E} + \frac{1}{c}\vec{v}_k \times \vec{B}\right) \\ \vec{v}_k = \frac{1}{\hbar}\nabla_k \varepsilon_k = 2\alpha k_x \hat{x} + 2\beta k_y \hat{y} \end{cases}$$

that is $\hbar \begin{pmatrix} \dot{k}_x \\ \dot{k}_y \\ \dot{k}_z \end{pmatrix} = -\frac{eB_z}{c} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2\alpha k_x & 2\beta k_y & 0 \\ 0 & 0 & 1 \end{vmatrix} = -\frac{2eB_z}{c}\begin{pmatrix} \beta k_y \\ -\alpha k_x \\ 0 \end{pmatrix}$

$$\begin{cases} \dot{k}_x = -\frac{2eB_z\beta}{c\hbar} k_y \\ \dot{k}_y = +\frac{2e\alpha B_z}{c\hbar} k_x \\ \dot{k}_z = 0 \end{cases} \Rightarrow \begin{cases} \ddot{k}_x = -\frac{2eB_z\beta}{c\hbar}\dot{k}_y = -\left(\frac{2eB_z}{c\hbar}\right)^2 \alpha\beta k_x \\ \ddot{k}_y = -\left(\frac{2eB_z}{c\hbar}\right)^2 \alpha\beta k_y \end{cases} \Rightarrow \begin{array}{l} k_x = A\sin(wt+\gamma_x) \\ k_y = B\sin(wt+\gamma_y) \end{array}$$

So $w^2 = +\left(\frac{2eB_z}{c\hbar}\right)^2 \alpha\beta$ $\qquad w = \frac{2eB_z}{c\hbar}\sqrt{\alpha\beta}$ $\qquad +20$

In real space, ~~$v_x = 2\alpha A\sin(wt+\beta)$~~ ~~$v_y = 2\beta B\sin(wt$~~

Since $\varepsilon = (\alpha k_x^2 + \beta k_y^2) = A^2\alpha \sin^2(wt+\gamma_x) + B^2\beta\sin^2(wt+\gamma_y) = \text{const}$

choose $A^2 = \frac{1}{\alpha}$ $\quad B^2 = \frac{1}{\beta}$ $\quad \gamma_y = \frac{\pi}{2}$ $\quad \gamma_x = 0$

So $\begin{cases} k_x = \frac{1}{\sqrt{\alpha}}\sin wt \\ k_y = \frac{1}{\sqrt{\beta}}\cos wt \end{cases}$

In real space $\begin{cases} v_x = 2\sqrt{\alpha}\sin wt \\ v_y = 2\sqrt{\beta}\cos wt \end{cases} \Rightarrow \begin{cases} x = -\frac{2\sqrt{\alpha}}{w}\cos wt + C_1 \\ y = +\frac{2\sqrt{\beta}}{w}\sin wt + C_2 \end{cases}$ $\qquad +13$

Orbit: $\frac{(x-C_1)^2}{\alpha} + \frac{(y-C_2)^2}{\beta} = 1 \times \frac{w}{4}$ ~ ecllipse